Strong Light-Induced Negative Optical Pressure Arising from Kinetic Energy of Conduction Electrons in Plasmon-Type Cavities

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We found that very strong negative optical pressure can be induced in plasmonic cavities by LC resonance. This interesting effect could be described qualitatively by a Lagrangian model which shows that the negative optical pressure is driven by the internal inductance and the kinetic energy of the conduction electrons. If the metal is replaced by perfect conductors, the optical pressure becomes much smaller and positive.

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The fact that electromagnetic (EM) radiation exerts pressure on any surface was deduced theoretically by Maxwell in 1871, and demonstrated experimentally by Lebedev in 1900 [1] and by Nichols and Hull in 1903 [2]. Recently, there has been increasing interest in optical forces acting on resonance cavities [3–7], partly due to the emergence of the field of cavity optomechanics [6–10]. One may anticipate that the radiation field confined by a cavity should serve to expand the cavity as the photons bounce back and forth between the cavity walls. This is indeed the case for a large class of resonant cavities, such as closed metallic rectangular cavities made up of perfect electric conductor (PEC) walls, whispering-gallery mode cavities, and Fabry-Perot cavities. We will show that a very simple system can display light-induced resonance that is not only large in magnitude, but also attractive rather than repulsive. This will provide a new platform to realize giant optical forces, in addition to micro- or nanosystems such as photonic crystals [11–13], plasmonic structures [14–17], microcavities and waveguides [18–20].

We propose that instead of the usual positive pressure, one can induce a strong negative pressure between the walls of an open metallic cavity, by utilizing the kinetic energy of the electrons inside the metallic cavity walls. Consider a planar structure consisting of a square metal patch placed above another bigger square metal slab, as illustrated in Fig. 1. The thickness of the patch and slab is $t = 10$ nm. The size of slab is taken to be 5 times of the patch, and the results are qualitatively the same as long as the slab is a few times the size than the patch. The distance between the patch and the slab is D. Metamaterials [21–26] exhibit artificial resonant responses, and such responses can be realized in the visible or infrared frequency range based on LC resonance effect [27–30]. Here, the patch and slab forms an LC resonance cavity [31]. We propose a simple Lagrangian model to describe qualitatively the resonance property of the equivalent LC circuit for this structure [32–35]. The circuit's Lagrangian could be expressed as [36].

$$\mathcal{L} = \frac{L\dot{Q}^2}{2} - \frac{Q^2}{2C}. \tag{1}$$

Here, $L = L_m + L_e$ and $C = C_m + C_e$, $L_m = \mu_0 D/2$ and C_m are the external inductance and capacitance, $L_e = 1/(\omega_p^2 \varepsilon_0 t)$ and C_e are the internal inductance and capacitance [37–39], Q is the net charge in the capacitor, $L_m\dot{Q}^2/2$ is the magnetic field energy stored between the two patches, $U_{\text{kinetic}} = L_e\dot{Q}^2/2$ is the kinetic energy of the electrons due to the induced current inside the metal. We will ignore the internal capacitance which is much smaller than external capacitor, i.e., $C \approx C_m$. Then $U_{\text{electric}} = Q^2/(2C_m)$ is the potential energy stored in the cavity. For a finite sized patch, the edge effect due to field leakage on both ends of the patch has to be considered, and C_m is modeled as $C_m = \varepsilon_0 \frac{2}{\pi^2} \frac{A^2}{D}(1 + \alpha \frac{D}{A})$ [36], where the empirical coefficient α gives the magnitude of edge effect.

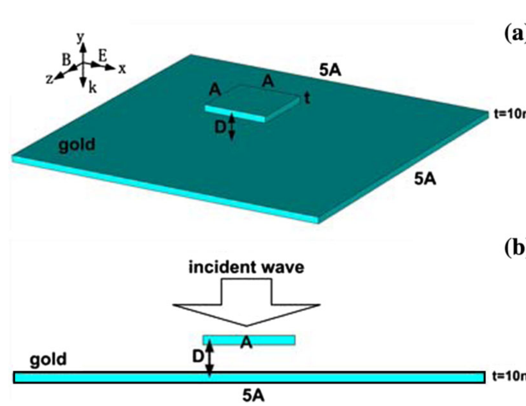

FIG. 1 (color online). Schematic of nanocavity with two gold patches.

In the presence of Ohmic dissipation and an external driving field, the Euler-Lagrange equation can be written as $\frac{d}{dt}(\frac{\partial \mathcal{L}}{\partial \dot{Q}}) - \frac{\partial \mathcal{L}}{\partial Q} = -\frac{\partial \mathcal{R}}{\partial \dot{Q}} + \text{e.m.f.}$, where $\mathcal{R} = R_{\text{eff}} \dot{Q}^2/2$ (R_{eff} is the effective resistance in the system) and the e.m.f. due to external field E_0 is $\text{emf} = (-\oint E d\ell) = -(\ell_{\text{eff}}^{(1)} E_0 - \ell_{\text{eff}}^{(2)} E_0 e^{ikD})$ with $\ell_{\text{eff}}^{(1)}$ and $\ell_{\text{eff}}^{(2)}$ being the effective length of the dipole on the patch and the slab, respectively, and the factor $\exp(ikD) = \exp(i\frac{\omega}{c}D)$ represents the retardation effect. To simplify expressions, we introduce the notation $\tau = \ell_{\text{eff}}^{(2)}/\ell_{\text{eff}}^{(1)}$ [36]. Solving the Euler-Lagrange equation gives the resonance frequency of the cavity $\omega_0 = \frac{1}{\sqrt{(L_e + L_m)C_m}} = \frac{1}{\sqrt{(\frac{1}{\varepsilon_0 \omega_p^2 t} + \frac{\mu_0 D}{2})\varepsilon_0 \frac{2}{\pi^2}\frac{A^2}{D}(1+\alpha\frac{D}{A})}}$. The optical force exerted on the patches can be calculated as a generalized force corresponding to the coordinate D [36]:

$$\mathcal{F} = \frac{\partial \mathcal{L}}{\partial D} = \frac{1}{2}\left(\frac{\partial L_m}{\partial D} + \frac{\partial L_e}{\partial D}\right)\dot{Q}^2 + \frac{1}{2}\frac{Q^2}{C_m^2}\left(\frac{\partial C_m}{\partial D}\right). \quad (2)$$

As the internal inductance does not depend on the gap distance, we have $\partial L_e/\partial D = 0$ [37–39]. Approximating our system by a parallel plate transmission line, we have, $L_m \sim D \Rightarrow \partial L_m/\partial D = L_m/D$. We also have $\frac{\partial C_m}{\partial D} = -\frac{C_m}{D}\frac{A}{A+\alpha D}$. At the resonance frequency ω_0, if time averaging is performed for one oscillation period, one could obtain $L\langle \dot{Q}^2\rangle_t = (L_m + L_e)\omega_0^2 \langle Q^2\rangle_t = \langle Q^2\rangle_t/C_m$. Substituting these into Eq. (2), one obtains

$$\langle \mathcal{F}\rangle_t = -\frac{1}{2}\frac{L_e\langle \dot{Q}^2\rangle_t}{D} + \frac{1}{2}\frac{\langle Q^2\rangle_t}{C_m}\frac{1}{D}\left(\frac{\alpha D}{A + \alpha D}\right). \quad (3)$$

Here, $\langle\cdot\rangle_t$ denotes the time averaging. We can see that the total optical force given in Eq. (3) includes a term $-\frac{1}{2}\frac{L_e\langle \dot{Q}^2\rangle_t}{D}$ which is the attractive force from kinetic energy of electron and a term $\frac{1}{2}\frac{\langle Q^2\rangle_t}{C_m}\frac{1}{D}(\frac{\alpha D}{A+\alpha D})$ which is the repulsive force from E field edge effect. If the edge effect is small, the optical force will be dominated by an attractive force $-\frac{1}{2}\frac{L_e\langle \dot{Q}^2\rangle_t}{D}$ derived from kinetic energy of the electron in the induced current. At the resonance frequency $\omega_0 = \frac{1}{\sqrt{(L_m+L_e)C_m}}$ $(k_0 = \frac{\omega_0}{c})$, $\frac{1}{2}\frac{\langle Q^2\rangle_t}{C_m} = \frac{1}{2}\frac{(L_m+L_e)\langle \dot{Q}^2\rangle_t}{D}$, $\langle \dot{Q}^2\rangle_t = (\frac{\ell_{\text{eff}}^{(1)}}{R_{\text{eff}}})^2[\frac{1+\tau^2}{2} - \tau\cos(k_0 D)]E_0^2$. The effective length of patch could be approximately taken as $\ell_{\text{eff}}^{(1)} \simeq A$, and the area of patch is $S = A^2 \simeq (\ell_{\text{eff}}^{(1)})^2$. The optical pressure normalized by the incident intensity $I_0 = \frac{1}{2}\varepsilon_0 c E_0^2$ could be obtained as [36] $P = \frac{\eta}{D}[\frac{1+\tau^2}{2} - \tau\cos(k_0 D)](-1 + (1 + \beta D)\frac{\alpha D}{A+\alpha D})$, where $\eta = \frac{t}{c}\frac{\omega_p^2}{\gamma_m^2}$, $\beta = \frac{t}{2}\frac{\omega_p^2}{c^2}$. Equation (3) suggests that the optical pressure between the the patch and the slab is induced by the LC resonance is determined by competition between negative pressure from the kinetic energy of electrons and positive pressure from the edge effect of E field. In the following numerical simulations, we will show that the negative pressure from kinetic energy is much stronger than the positive pressure from edge effect. Then the total optical pressure P is negative.

We shall now proceed by using numerical simulation with a commercial software package CST MICROWAVE to show that our Lagrangian model can capture the essence of physics and a giant negative pressure can indeed be induced by external light. In the simulations, the metal permittivity gold is taken to be of the Drude form $\varepsilon(\omega) = 1 - \frac{\omega_p^2}{(\omega^2 + i\gamma_m\omega)}$, with $\omega_p = 1.37 \times 10^{16}\ \text{s}^{-1}$ and $\gamma_m = 12.08 \times 10^{13}\ \text{s}^{-1}$ for gold. These characteristic frequencies are fitted from experimental data [40]. For the structure depicted in Fig. 1, the parameters are chosen as $A = 200$ nm, $D = 30$ nm and $t = 10$ nm in the simulations. Open boundary conditions are used in all three directions. The incident plane EM propagates upward along the y direction. The E field of incident wave is set as $E_0 = 1.0$ V/m. When the incident frequency is swept from 150 to 350 THz, the frequency dependence of E field between two patches is plotted in Fig. 2(a). One LC resonance mode is excited at 255 THz. The profiles of the electric and magnetic field at the LC resonance mode are shown in Fig. 2(c) and 2(d). We see that the field strongly localizes in the space between the patch and the slab at this resonance frequency.

After obtaining the EM fields, the time-averaged optical force $\langle \mathcal{F}\rangle_t$ between the patch and slab can then be calculated rigorously using the Maxwell's stress tensor via a surface integral $\langle \mathcal{F}\rangle_t = \oint_s \langle T\rangle_t ds$, where $T_{\alpha\beta} = \varepsilon_0(E_\alpha E_\beta - EE\delta_{\alpha\beta}/2) + \mu_0(H_\alpha H_\beta - HH\delta_{\alpha\beta}/2)$ and S is the integration surface enclosing one patch. Figure 2(b) shows the calculated optical force per unit area acting on the surface of patch as a function of frequency. At the resonance frequency, a strong negative (attractive) pressure on the wall of cavity could be obtained (about $-698\ \text{Pa}/(\text{mW}/\mu\text{m}^2)$). For an infinite perfectly reflecting plate, the positive optical pressure exerted by a plane wave

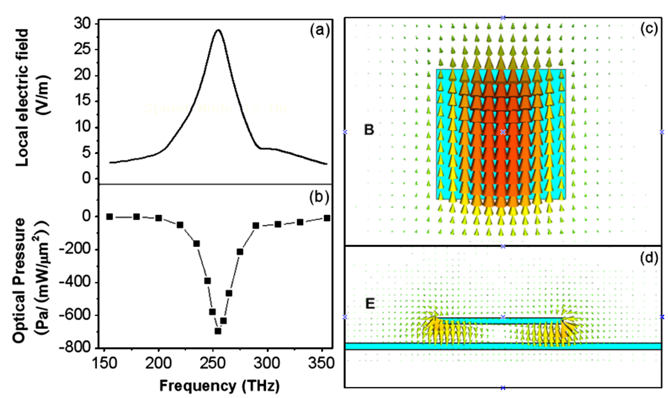

FIG. 2 (color online). The frequency dependence of (a) local electric field between two patches and (b) optical pressure between two patches (with $A = 200$ nm, $D = 30$ nm); (c) Magnetic field (on y-cut middle layer) and electric field (on z-cut middle layer) at the resonance frequency $\omega = 255$ THz.

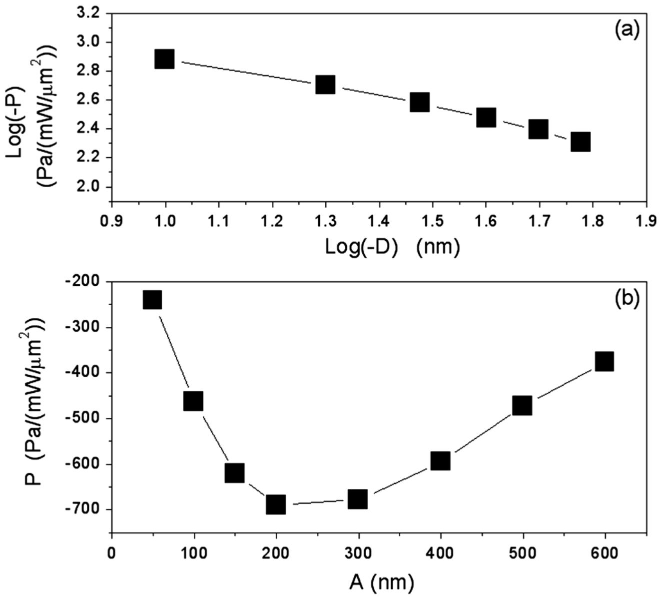

FIG. 3. (a) The log-log relationship between optical pressure P and distance D (with $A = 600$ nm); (b) the dependence of optical pressure P on the size parameter A (with $D = 30$ nm).

with the same frequency is only about 10 $\mathrm{Pa}/(\mathrm{mW}/\mu\mathrm{m}^2)$. The negative optical pressure in the plasmonic cavity is much stronger than the usual optical pressure. Such a strong negative optical pressure might be able to mechanically deform the structure, leading to optomechanical coupling of the photon and phonon modes [6–10]. The direction of the force is opposite to the propagating direction of the incident light [1,2].

The dependence of the optical pressure on the distance D was numerically simulated and plotted in Fig. 3(a). For small D ($D \ll A$), as the edge effect term in Eq. (3) can be neglected, P is dominated by the kinetic energy term, which carries an explicit $1/D$ dependence. Consequently, the log of $(-P)$ should scale linearly with $\log(D)$ for small D. The log-log relationship between optical pressure P and distance D is given as black square dots in Fig. 3(a). As shown in Fig. 3(b), for the small distance D below 50 nm, $\log(-P)$ is indeed inversely proportional to $\log(D)$. For D greater than 50 nm, the edge effect term in Eq. (3) cannot be ignored and its positive pressure will counteract part of the negative pressure from the kinetic energy term. As a result, the total optical pressure is no longer inversely proportional to D and the calculated $\log(-P)$ deviates from a straight line progressively as D increases, as shown in Fig. 3(a). In our CST simulations, the dependence of the optical pressure on the parameter A was also investigated and plotted in Fig. 3(b). There is an optimal patch size that gives a maximum force, and the optical pressure decreases for smaller and bigger patches. We show in the Appendix [34] that the salient features of the numerical calculations can be reproduced with the Lagrangian model. In particular, the Lagrangian model gives a simple picture to show that the optical pressure in plasmonic cavity is mainly driven by the near-field localized LC resonance coupling and the kinetic energy inside the plasmonic plates is the key behind the strong negative pressure.

To further clarify the role of the kinetic energy of electrons, we consider the same structure illustrated in Fig. 1 with $A = 200$ nm and $D = 30$ nm, but with all metals replaced by perfect conductors (PEC). Since the EM field does not penetrate into the PEC, there is no contribution from the kinetic energy of the electrons. According to our model, the optical pressure only comes from the edge effect term in Eq. (3), which should be repulsive and small as A is much bigger than D. Indeed, the numerical calculated optical force for the PEC structure is only 187 $\mathrm{Pa}/(\mathrm{mW}/\mu\mathrm{m}^2)$, much smaller than the attractive force of -698 $\mathrm{Pa}/(\mathrm{mW}/\mu\mathrm{m}^2)$ in the plasmonic structure (see Table I). Moreover, the optical pressure for the PEC structure is positive, contrary to the negative pressure in plasmonic systems. While the numerical calculations are fully electrodynamical, we can understand the results qualitatively by considering the quasistatic limit, as we have a subwavelength system. In that limit, the optical force can be separated into a Coulomb electric force $\langle \mathcal{F}_e \rangle_t = \oiint \langle T^e_{\alpha\beta} \rangle_t ds$ and an Ampere magnetic force $\langle \mathcal{F}_m \rangle_t = \oiint \langle T^m_{\alpha\beta} \rangle_t ds$, where $T^e_{\alpha\beta} = \varepsilon_0 (E_\alpha E_\beta - EE\delta_{\alpha\beta}/2)$ and $T^m_{\alpha\beta} = \mu_0 (H_\alpha H_\beta - HH\delta_{\alpha\beta}/2)$. Since the former is induced by charges and the latter by current, we can write $\langle \mathcal{F}_e \rangle_t = \oiint \langle T^e_{\alpha\beta} \rangle_t ds = -\frac{\langle Q^2 \rangle_t}{C} \frac{1}{D}$ and $\langle \mathcal{F}_m \rangle_t = \oiint \langle T^m_{\alpha\beta} \rangle_t ds = \frac{L_m}{D} \langle \dot{Q}^2 \rangle_t$. As the size of the patch is finite, the E field at the edge of the patch cannot be completely confined under the patch. Table I shows that this edge effect of E field makes the attractive Coulomb force -299.5 $\mathrm{Pa}/(\mathrm{mW}/\mu\mathrm{m}^2)$smaller than the repulsive Ampere force (487.3 $\mathrm{Pa}/(\mathrm{mW}/\mu\mathrm{m}^2)$), and the difference

TABLE I. Numerically calculated optical forces for the plasmonic and PEC structure (with $A = 200$ nm, $D = 30$ nm) at resonance frequencies.

Material	Optical force calculated from magnetic field $[\mathrm{Pa}/(\mathrm{mW}/\mu\mathrm{m}^2)]$ $\langle \mathcal{F}_m \rangle_t = \oiint \langle T^m_{\alpha\beta} \rangle_t ds$	Optical force calculated from electric field $[\mathrm{Pa}/(\mathrm{mW}/\mu\mathrm{m}^2)]$ $\langle \mathcal{F}_e \rangle_t = \oiint \langle T^e_{\alpha\beta} \rangle_t ds$	Total optical pressure $[\mathrm{Pa}/(\mathrm{mW}/\mu\mathrm{m}^2)]$ $\langle \mathcal{F} \rangle_t = \oiint [\langle T^e_{\alpha\beta} \rangle_t + \langle T^m_{\alpha\beta} \rangle_t] ds$	Optical Pressure
Drude	151	−849	−698	Negative
PEC	487.3	−299.5	187.9	Positive

of 187.9 Pa/(mW/μm^2) is the residual positive pressure due to edge effect as described in the Lagrangian model. However, the situation is entirely different for the plasmonic cavity. In the structure made of Drude metal and at frequencies in which the field penetrates the metal, most of the inductance is manifested as the kinetic energy of the electrons and only a small part in the magnetic field. The electric field energy becomes stronger than the magnetic field energy, and this difference induces strong optical forces in the plasmonic structure. In our simulated data given in Table I, the negative optical pressure from the electric field (− 849 Pa/(mW/μm^2)) is much larger than the positive optical pressure from the magnetic field (151 Pa/(mW/μm^2)). These results agree with our theoretical model quite well.

In the above discussion, we focused on the optical pressure in a single plasmonic cavity. For a periodic array of many such plasmonic cavities, the total optical force could be seen as sum of optical force from many such plasmonic resonators if the coupling interaction between them is small, and coherent coupling may further enhance the effect. This could produce a substantial optical force in an extended system. Not only is this optical force strong, the phenomenon is expected to be very robust. We can see from Fig. 2(b) that the quality factor is relatively low, at least when compared with, for example, whispering-gallery mode resonators. In principle, one could also obtain strong optical forces between two objects if high-fidelity resonances (such as whispering-gallery mode) are excited, but the frequency must be very precise in those "photonic molecules" and would be much more difficult to realize than the present configuration which employs plasmonic resonances. We note that in ordinary electromagnetic cavities, such as Fabry-Perot cavities or micro-disk resonators, the resonance force is repulsive. In our plasmonic system, the penetration of the field into the metal leads to a giant attractive optical force.

In summary, we designed a plasmonic cavity system comprising a patch and a slab. We find that the kinetic energy of conduction electrons plays a key role in inducing a strong negative optical pressure. A Lagrangian model is proposed to describe the salient features. The mechanism and theoretical model reported in this paper could have potential applications in many other subwavelength optomechanical plasmonic structures.

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