Heralded generation of multipartite entanglement for one photon by using a single two-dimensional nonlinear photonic crystal

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Abstract: We propose a compact scheme for the heralded generation of single-photon multipartite entanglement by using a single two-dimensional nonlinear photonic crystal. Studies have shown that by appropriate structure design, the single-photon entanglement shared among three spatially distinct optical modes can be generated through three concurrent spontaneous parametric down-conversion processes by using the other photon in an identical spatial mode as a trigger. Furthermore, we analyze the entanglement of such heralded single-photon tripartite W-type state theoretically. This method can be expanded for the heralded single-photon N-partite entanglement generation. This compact and stable quantum light source may act as a key ingredient in quantum information science.

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1. Introduction

Recently, the single-photon entangled state delocalized between two spatial modes has attracted extensive interests [1–7]. After demonstrating the fundamental concerns about the single-photon entanglement [1], nonlocality [2,3] and purification [4], people find interesting applications of single-photon entanglement in quantum teleportation [5,6] and quantum network to link two solid-state nodes [7]. We can design a scheme for heralded quantum entanglement between N quantum memories for long-distance quantum communication and scalable quantum networks [7–13] if we can generate single-photon N-mode entanglement. A consequent notion is to extend such single-photon two-mode entanglement into a multipartite entangled fashion by taking use of the advantage that the Hilbert space of spatial mode can be easily expanded, which will certainly improve the performance of quantum information processing like an entangled multi-particle system does [14].

A single-photon multipartite entanglement delocalized between multiple spatial modes belongs to the W-type entangled state [15,16], which is robust against loss [17]. Usually it can be prepared by using a couple of linear optical components from a single-photon source. Here we report a compact scheme for the single-photon N-mode entanglement generation directly from a single two-dimensional (2D) nonlinear photonic crystal (NPC), which contains a periodical variation of the second-order nonlinearity while the refractive index keeps constant [18,19]. Studies have shown that the NPC opens a new way to engineer the spatial entanglement either resulting in new types of path-entangled states [20] or transform the twophoton wavefront [21,22] with the unique quasi-phase-matching (QPM) technique. Here, in this work, by appropriate structure design in a single 2D NPC, we show the single-photon entanglement shared among N (N = 2,3,4) spatially distinct optical modes can be generated through N concurrent QPM spontaneous parametric down-conversion (SPDC) processes while the other photon sharing an identical spatial mode is used as the trigger. The detection of the heralded single-photon entanglement can refer to the uncertainly relations [15,16] in an interferometric setup [23]. The multipartite entanglement is ensured by the coherent beamsplitting of single-photon inside the NPC, which indicates such a method to be compact

and stable, hence can act as a useful quantum light source for integrated quantum information processing. This method is extendable to any high-dimensional mode-entanglement based on the principle of structure design.

2. Theoretic analysis

In this work, the designed 2D NPC follows a hexagonally sequence as shown in Fig. 1(a), where circularly inverted domains (with $-\chi^{(2)}$) distribute periodically on a + $\chi^{(2)}$ background with a period of Λ . First, we investigate the structure design and theoretically characterize the single-photon tripartite entanglement. In Fig. 1(b), three reciprocal vectors $\vec{G}_{m,n} = m\vec{e}_1 + n\vec{e}_2$, (*m*, *n* are integers) are designed to simultaneously work for different SPDC processes,

The paired file designed to simulationary work for different SFDC processes, respectively. We consider the pump is incident along z-axis and the photon-pair is nondegenerate. Figure 1(c) is the transverse pattern of parametric light in the Fourier plane. The paired tilted reciprocal vectors ensure the photon-pair to emit noncollinearly as four conical beams. It is notable that the photon-pair either propagates along the left two conical beams or the right two conical beams in a superposed way as shown in Fig. 1(c), which is not available from the birefringence-phase-matching (BPM) crystals. For the degenerate photon-pair, the signal and idler conical beams are overlapped. The reciprocal vector along the z-axis ensures a collinear SPDC and the signal photon and idler photon both emit along z-axis in a beam-like way. So for a pair of photons, due to three concurrent SPDC processes, when the idler photon is captured within the overlapped region along z-axis, the signal photon should be shared indistinguishably among three optical modes a, b and c. For the single-photon two-mode entanglement, the structure design is rather simple. It requires that only a pair of tiled reciprocal vectors take the role in the SPDC processes, which can be easily designed in a common 2D NPC.



Fig. 1. (a) Sketch of a hexagonal poled 2D NPC. (b) Reciprocal lattice of the crystal and the geometries of three concurrent QPM SPDC processes. (c) Transverse pattern of the parametric light in the Fourier plane.

When the crystal size is considered infinitely large, the nonlinear susceptibility $\chi^{(2)}$ can be written as a Fourier series

$$\chi^{(2)}(\vec{r}) = d_{eff} \sum_{m,n} f_{m,n} e^{-i\vec{G}_{m,n}\cdot\vec{r}},$$
(1)

where d_{eff} is the effective nonlinear coefficient and $\vec{r} = (y, z)$ is the 2D spatial coordinate. The Fourier coefficient takes the form of $f_{m,n} = 8\pi (l/\Lambda)^2 \times J_1(|\vec{G}_{m,n}|l)/(\sqrt{3}\times|\vec{G}_{m,n}|l)$. J_1 is the first order Bessel function and $|\vec{G}_{m,n}| = 4\pi \sqrt{m^2 + n^2 + mn}/(\sqrt{3}\Lambda)$. The $l, l/\Lambda$ are the

radius of the circularly inverted domain and the duty cycle, respectively. Using multi-mode description of SPDC processes, we obtain the interaction Hamiltonian inside such 2D NPC [20]

$$\hat{H}_{I}(t) = \varepsilon_{0} \int_{V} d^{3}\vec{r} \chi^{(2)}(\vec{r}) \vec{E}_{p}^{(+)}(\vec{r},t) \hat{E}_{s}^{(-)}(\vec{r},t) \hat{E}_{i}^{(-)}(\vec{r},t) + H.c.,$$
(2)

where V denotes the interaction volume. The pump field is assumed to be plane wave while the signal and idler fields are represented by quantum operators. By using the first-order perturbation approximation in the interaction picture, we obtain the two-photon state as

$$\left|\Phi\right\rangle = A_{1}\int d\nu\Psi_{1}(\nu)\left|\omega_{s_{1}}\omega_{i_{1}}\right\rangle_{ab} + A_{2}\int d\nu\Psi_{2}(\nu)\left|\omega_{s_{2}}\omega_{i_{2}}\right\rangle_{bb} + A_{3}\int d\nu\Psi_{3}(\nu)\left|\omega_{s_{3}}\omega_{i_{3}}\right\rangle_{bc}, (3)$$

in which the coefficient $A_j = d_{33}Lf_{m_j,n_j}\sqrt{\Omega_{s_j}\Omega_{i_j}/(n_{s_j}n_{i_j})} \times iE_p/(2c)$, (j = 1,2,3), and we have $A_1 = A_3$. E_p is the electrical field amplitude of pump and $n_{s(i)}$ denotes the refraction index of an e-polarized photon at central frequency $\Omega_{s(i)}$. The detuning variable v is the bandwidth of the down-converted light and $|v| \ll \Omega_{s(i)}$. The joint spectrum density determines the bandwidth of two-photon state and takes the following form

$$\Psi_{j}(\nu) = \frac{1 - e^{-i\Delta k_{z_{j}}(\nu)L}}{i\Delta k_{z_{j}}(\nu)L} = e^{\frac{-i\Delta k_{z_{j}}(\nu)L}{2}} \operatorname{sinc}(\frac{\Delta k_{z_{j}}(\nu)L}{2}),$$
(4)

where *L* is the length of NPC along z direction and $\Delta k_{z_j}(v)$ is the phase mismatching along z direction for the different SPDC process denoted by *j*. For the noncollinear type-0 nondegenerate QPM SPDC, the joint spectrum density can be written as $|\Psi_{1(3)}(v)| = |\sin c(vL(\cos\theta/u(\Omega_s) - 1/u(\Omega_i))/2)|$, while under the collinear situation, $|\Psi_2(v)| = |\sin c(vL(1/u(\Omega_s) - 1/u(\Omega_i))/2)|$, where $u(\Omega_{s(i)})$ is the group velocity of parametric photon at $\Omega_{s(i)}$ and θ is the angle between the signal and pump beams. After taking the single-frequency approximation, we can write the output entangled state as

$$\left|\Phi\right\rangle = A_{\rm I}\left(\left|\Omega_{s,a}\Omega_{i,b}\right\rangle + \left|\Omega_{s,b}\Omega_{i,c}\right\rangle\right) + A_{\rm 2}\left|\Omega_{s,b}\Omega_{i,b}\right\rangle. \tag{5}$$

To show the experimental feasibility of our scheme, we design a 2D periodic poled lithium niobate (PPLN) crystal for simultaneous realization of three QPM SPDC processes at $440.36 \text{ nm} \rightarrow 810 \text{ nm} + 964.97 \text{ nm}$ following the $e \rightarrow e + e$ configuration. $|\vec{G}_{5-5}| = |\vec{G}_{0.5}| = 20\pi/(\sqrt{3}\Lambda)$ ensure the QPM condition $(\vec{k}_p = \vec{k}_s + \vec{k}_i + \vec{G}_{m,n})$ in the noncollinear way for the conical emission of the photon-pair, while $|\vec{G}_{20}| = 8\pi/(\sqrt{3}\Lambda)$ is designed to match the collinear SPDC process. Setting working temperature as 180 °C, we get the modulation period $\Lambda = 8.14 \ \mu m$. The angle between $\vec{k}_{s,a(c)}$ and \vec{k}_p is $\theta_{I(2)} = 13.17^{\circ}$ (smaller than the total reflection angle of about 27°). We can quantify this three-body pure state by using distributed entanglement [17,24]. $C_{a,b(c)}$ means the concurrence between a and b(c), $C_{a,bc}$ means the concurrence between a and bc. We can obtain $C_{a,b,c} = 2 \left| A_1 A_{2(3)} / (A_1^2 + A_2^2 + A_3^2) \right|, \ C_{a,b,c} = 2 \left| A_1 \sqrt{A_2^2 + A_3^2} / (A_1^2 + A_2^2 + A_3^2) \right|.$ The

entanglement quality is affected by several experimental parameters, like the duty cycle (l/Λ) . We have made such calculations as shown in Fig. 2. We can get the maximally

entangled state when $l/\Lambda \approx 0.0001, 0.2475, 0.2704, 0.4726$, which corresponds to the case of $A_1 = A_2$, and $C_{a,b(c)} = 2/3$, $C_{a,bc} = 2\sqrt{2}/3$.



Fig. 2. (a) The concurrence $C_{a,bc}$ varies with the duty cycle of 2D PPLN crystal. (b) The concurrence $C_{a,bc}$ varies with the duty cycle of 2D PPLN crystal.

The setup for achieving a heralded single-photon tripartite entanglement is shown in Fig. 3. The entangled photon-pair is nondegenerate and we use a dichroic mirror to separate the idler photon from the signal photon after the pump photons are suppressed. When the idler photon is triggered, we get a heralded single-photon three-mode entangled state as the following

$$|\Phi\rangle_{abc} = A_1 \eta_1 (|100\rangle_{abc} + |001\rangle_{abc}) + A_2 \eta_2 |010\rangle_{abc}, \qquad (6)$$

where η_1 and η_2 are the collection efficiency for the signal photon belonging to the two symmetric non-conlinear SPDC processes and the conlinear SPDC process, respectively. For the W-type maximally entangled state generation, we should design $A_1\eta_1 = A_2\eta_2$. Since η_1 and η_2 can both be altered by different experimental conditions, like the temperature and the coupling technique, we can alter η_1 and η_2 to achieve the necessary condition when the duty cycle is not well achieved.



Fig. 3. Schematic for single-photon tripartite W state generation and entanglement analysis. IF, interference filter; DM, dichroic mirror. Beam-splitter setup to project onto the three $|W_j\rangle$ states: 3 input modes are converted into 3 output modes by 3 lossless beamsplitters. The lowermost beam splitter has a reflectivity R = 1/3, for the other two R = 1/2.

For simple experimental tests, we can use uncertainty relations as the entanglement criterion to verify our single-photon tripartite entanglement [15,16]. The mode transformation from $\{|100\rangle, |010\rangle, |001\rangle\}$ to $\{|W_j\rangle = (|100\rangle + e^{2\pi(j-1)/3}|010\rangle + e^{4\pi(j-1)/3}|001\rangle)/\sqrt{3}\}$, (j = 1,2,3) can be easily decomposed in terms of unitary operations that can be implemented with beamsplitters and phaseshifters (see Fig. 3) [23]. We choose three projectors onto the basis $|W_j\rangle$ as nonlocal observables $M_j = |W_j\rangle < W_j$. For any state ρ

$$\Delta \rho = \sum_{j=1}^{3} \left\{ Tr(\rho[|W_j\rangle \langle W_j|]^2) - (Tr\rho[|W_j\rangle \langle W_j|])^2 \right\}.$$
⁽⁷⁾

The single-photon tripartite entanglement is three-body pure state for the relative phase between any two modes is fixed in a single crystal wafer. So for maximal three-mode entanglement, $\Delta \rho = 0$ and for partial three-mode entanglement, $0 < \Delta \rho < 1/2$.

Furthermore, we design a structure appropriate for the generation of four-partite entanglement from a hexagonally domain-inverted PPLN crystal. The involved reciprocal vectors are $|\vec{G}_{2,-1}| = |\vec{G}_{-1,2}| = 4\pi/\Lambda$, $|\vec{G}_{6,-4}| = |\vec{G}_{-4,6}| = 8\sqrt{7}\pi/(\sqrt{3}\Lambda)$. As shown in Fig. 4, for 519.1 nm \rightarrow 810 nm+1445.4 nm, four type-0 QPM SPDC processes can happen simultaneously. The working temperature is set as 180°C. The modulation period $\Lambda = 5.85 \ \mu\text{m}$. The angle between $\vec{k}_{s,a(d)}$ and \vec{k}_p is 21.45° and the angle between $\vec{k}_{s,b(c)}$ and \vec{k}_p is 6.31°. Using the uncertainty relations mentioned above, we can further theoretically characterize this heralded single-photon four-partite W-state. In principle, for any singlephoton N-partite (N>4) mode-entanglement, it is achievable when the cascaded 1D or 2D domain structures are engineered in a single crystal wafer. In this case, the phase of spatial modes from different domain-structure may differ, then each component of W-state may contain a phase term $(A_1 | 10...0 \rangle + e^{i\varphi_1} A_2 | 01...0 \rangle + ... + e^{i\varphi_{(N-1)}} A_N | 00...1 \rangle$), but the relative phase between any two modes are fixed, so the single-photon W-state from a single wafer is a pure state.



Fig. 4. (a) Transverse pattern of the parametric light in the Fourier plane. (b) Reciprocal lattice of the crystal and the geometries of four concurrent QPM SPDC processes.

3. Conclusion

In summary, we have proposed a compact scheme for single-photon N-partite (N = 2,3,4) entanglement by utilizing the flexible QPM technique. The N concurrent SPDC processes act as (N-1) coherent beamsplitters to distribute the single-photon into N distinct optical modes, which ensures the generated N components can be maximally entangled in a W-state not partially entangled. The method is extendable to any N-mode entanglement generation when the cascaded 1D or 2D structure on a single wafer is adopted. Such compact and stable N-partite entangled state with high generation rate can have potential applications in quantum fundamental tests and quantum repeater for quantum networks.

Compared with the conventional bi-particle entanglement such as polarization or time-bin entanglement, there are mainly two advantages of our single-photon mode-entangled source. Firstly, we can improve the efficiency of many protocols about quantum repeaters and quantum networks by the heralded generation of multipartite entanglement for one photon. Secondly, single-photon multi-mode W-type entangled source is the more economy resource

than bi-particle entanglement and we can improve the performance of quantum information processing using single-photon N-mode entangled source like an entangled multi-particle system does. On the other hand, though our single-photon N-mode W-type entangled state is more robust to loss than other type entangled state, this type of entanglement is also sensitive to decoherence inevitably during the practical transmission of long-distance quantum communication and quantum networks. We can "repair" entanglement by using entanglement purification technique [4,25–29].

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